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# PREVE: A Policy Recommendation Engine based on Vector Equilibria Applied to Reducing LeT's Attacks

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***Abstract*—We consider the problem of dealing with the terrorist group Lashkar-e-Taiba (LeT), responsible for the 2008 Mumbai attacks, as a five-player game. However, as different experts vary in their assessment of players' payoffs in this game (and other games), we identify multi-payoff equilibria through a novel combination of vector payoffs and well-supported $\epsilon$-approximate equilibria. We develop a grid search algorithm for computing such equilibria, and provide experimental validation using three payoff matrices filled in by experts in India-Pakistan relations. The resulting system, called PREVE, allows us to analyze the equilibria thus generated and suggest policies to reduce attacks by LeT. We briefly discuss the suggested policies and identify their strengths and weaknesses.**

## I. Introduction

Virtually all past work on counter-terrorism policy is qualitative (see [1] for an overview). A group of experts gather around a table, hypothesize about the impacts of different possible policies, and then decide which one to use. It is only recently that quantitative methods for generating policies against terror groups have started playing a role. Data mining approaches have been used to study the Pakistani terror group Lashkar-e-Taiba [2] with considerable impact in the strategic policy community in both the US and India, both of whom have attended talks on the results.

However, when it comes to the application of game-theoretic reasoning to international strategic elements [3] with both state and non-state actors, the situation becomes much more complex because identifying the "payoffs" for different players is an enormous challenge and experts vary widely on what these payoffs are. Finding equilibria with a single payoff matrix [3] in such situations poses an enormous challenge, since even small changes to the payoffs can lead to widely varying Nash equilibria. Vector payoffs [4] serve as a more credible mechanism that can be used to model such situations as a single strategy (i.e., one action per player), taking into account multiple possible payoffs, one corresponding to each of a set of payoff matrices.

Informally, given a real number $\epsilon$ and a set $U$ of payoff matrices, a strategy profile is a *multiple payoff $\epsilon$-equilibrium* if and only if for each payoff matrix $M \in U$ and for each player $p$, player $p$'s strategy is an approximate best response to other players' strategies, i.e., player $p$ cannot gain more than $\epsilon$ by deviating from his or her *multiple payoff $\epsilon$-equilibrium* strategy. In other words, even though experts may disagree over payoffs by providing different payoff matrices $M \in U$, a multiple payoff $\epsilon$-equilibrium is guaranteed to be an approximate equilibrium w.r.t. each of these diverse payoff matrices, i.e., it is robust w.r.t. the variations in the different experts' payoff matrices.

An alternative incarnation that we also introduce allows us to consider only $n$ of the experts from a given pool of experts. Here, we may choose to ignore some experts who hold "outlier" positions. These are called $(\epsilon, n)$-equilibria for an integer $n \geqslant 1$. A strategy profile is a *multiple payoff $(\epsilon, n)$-equilibrium* if and only if it is a *multiple payoff $\epsilon$-equilibrium* for some subset $U'$ of payoff matrices $U$ and $|U'| = n$. With a user choosing $n$, it is possible to ignore payoff matrices provided by experts that are highly inconsistent with others.

The definitions given above are very general as the notion of multiple payoff $\epsilon$-equilibrium can be easily altered by simply replacing approximate equilibrium in the italicized sentence with specialized kinds of approximate equilibria, e.g. well-supported approximate equilibrium [5] or approximate Nash equilibria [6]. In fact, this paper looks (formally) at well-supported multiple $\epsilon$-approximate Nash equilibria, which are defined in Section II. The PREVE (Policy Recommendation Engine based on Vector Equilibria) software we have built allows us to identify such equilibria and study them.

In this paper, we build on [7], which introduced a five-player game with the goal of reducing attacks by the terrorist group Lashkar-e-Taiba (LeT) that carried out the infamous 2008 Mumbai attacks. However, because [7] used only one payoff matrix, there are questions about how the results would change if the payoff matrix changed. In contrast, PREVE is applied to this situation with the following significant improvements.

- First, we obtained payoff matrices from three experts in the politics of South Asia and LeT in particular—none had any background in game theory and none had ethnic origins in the Indian subcontinent, to avoid bias. Two were retired US State Department employees with over 30 years of knowledge of negotiations in the region. The third was the author of two well-known books on terrorism, including one dealing with Pakistan and Lashkar-e-Taiba in particular. The payoff matrices were created completely independently using *open source* information as well as expertise of these experts by following a set of instructions on what payoff values meant.

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TABLE I. THE ACTIONS THAT DIFFERENT PLAYERS CAN TAKE.

| Player | Action | Abbrv. |
|---|---|---|
| Lashkar-e-Taiba (LeT) | Launch major attacks | attack |
| | Eliminate armed wing | eaw |
| | Hold attacks | hold |
| | Do nothing | none |
| Pakistan's Government (PakG) | Prosecute LeT | pros |
| | Endorse LeT | endorse |
| | Do nothing | none |
| Pakistan's Military (PakM) | Crackdown on LeT | crack |
| | Cut support to LeT | cut |
| | Increase support to LeT | support |
| | Do nothing | none |
| India | Covert action against LeT | covert |
| | Coercive diplomacy against PakG | coerce |
| | Propose peace initiative to PakG | peace |
| | Do nothing | none |
| U.S. | Covert action against LeT | covert |
| | Cut aid to PakG | cut |
| | Expand aid to PakG | expand |
| | Do nothing | none |

TABLE II. ALL $(\epsilon, n)$ EQUILIBRIA WITH $\epsilon = 0, n = 2$ IN WHICH LET DOES NOT ATTACK.

| Equil. | LeT | PakG | PakM | India | US |
|---|---|---|---|---|---|
| $E^1_{0,1,3}$ | eaw | pros | crack | covert | cut |
| $E^2_{0,1,3}$ | eaw | pros | crack | 0.75: covert<br>0.25: coerce | cut |
| $E^3_{0,1,3}$ | eaw | none | crack | coerce | cut |
| $E^4_{0,1,3}$ | none | pros | support | covert | cut |
| $E^5_{0,2,3}$ | eaw | pros | crack | coerce | cut |
| $E^6_{0,2,3}$ | none | none | crack | covert | cut |

- Second, we introduced a variant of vector equilibria called $(\epsilon, n)$-equilibria which support both multiple payoff matrices and multiple players and also account for cases where equilibria may not exist. Though it builds on vector equilibria, it is considerably different.
- Third, we conducted a detailed analysis of the equilibria we discovered; only some of the results are presented here due to space constraints.

The five players considered by [7] are: the US, India, the Pakistani military, the Pakistani civilian government, and the terrorist group Lashkar-e-Taiba (LeT) that carried out the 2008 Mumbai attacks [8], [2]. Table I shows the actions the players were allowed to take.

We found no $(0, 3)$-equilibria where LeT did not perform violent actions, but we did find the following:

1) There were 20 $(0, 2)$-equilibria in which experts #1 and #3 agreed, 218 $(0, 2)$-equilibria with experts #2 and #3, and 14 $(0, 2)$-equilibria in which experts #1 and #2 agreed.
2) Of these 252 $(0, 2)$-equilibria, there were just six in which LeT did not carry out attacks. There were no $(0, 2)$-equilibria involving experts #1 and #2. Table II below summarizes the actions present in these six situations. An equilibrium named $E_{\epsilon,j,k}$ is used to denote an $(\epsilon, 2)$ equilibrium in which the two players who "agree" are $j, k$.
   *In all six $(0, 2)$-equilibria listed above where LeT stands down, the US cuts aid (development and military) to Pakistan, and India either carries out covert action against LeT or engages in coercive diplomacy*. Moreover, in most $(0, 2)$-equilibria, the Pakistani military must crack down on LeT (though there is one case where they may expand support) and additionally, the Pakistani government must mostly prosecute LeT leaders (though there are two cases where they could do absolutely nothing). When we look at experts #2 and #3, we see that there are only two $(0, 2)$-equilibria in which LeT does not attack—in one India takes covert action and the US cuts aid. In both scenarios, the Pakistani military cracks down on LeT—in one the Pakistani government prosecutes LeT personnel and does nothing in the other.
   When we do the same with experts #1 and #3, we see that there are four $(0, 2)$-equilibria in which LeT does not attack. In all four, India takes either covert action or applies coercive diplomacy and the US cuts aid. In three cases, LeT eliminates its armed wing, while in another it does nothing. In the other two, LeT has a 50% and 75% chance of doing nothing and a 50% (resp. 25%) chance of attacking. In three cases, the Pakistani government prosecutes LeT personnel and does nothing in the fourth. In three of the cases, the Pakistani military cracks down on LeT, and in the one remaining case, it actually expands support for LeT. What these results suggest is that *India should expand covert action against LeT with the US cutting financial aid to Pakistan at the same time* if the goal is to reduce violence by LeT.
3) We also looked at $(0.1, 2)$-equilibria, i.e. $\epsilon = 0.1$, which means that each player may lose up to 10% of their best utility while being near an equilibrium with 2 of the 3 experts. In this case, we see no $(0.1, 2)$-equilibria involving experts #1 and #2 where LeT does not attack. But with experts #1 and #3, and experts #2 and #3, we do see such equilibria. As all $(0, 2)$-equilibria continue to be $(0.1, 2)$-equilibria, we only show new (0.1,2)-equilibria in Table III.
   With $\epsilon = 0.1$, we only get three new equilibria as compared to Table II. In all of these, the US needs to cut aid to Pakistan and India needs to carry out covert action against LeT. As in the previous table, this requires that the Pakistani government prosecute LeT. Even with an expansion in Pakistani military support for LeT, this provides hope that covert action on India's part and cuts in US aid to Pakistan will lead to reduced terrorist attacks by LeT.

TABLE III. ALL $(\epsilon, n)$ EQUILIBRIA WITH $\epsilon = 0.1, n = 2$ WHICH ARE NOT $(0, 2)$-EQUILIBRIA, AND IN WHICH LET DOES NOT ATTACK.

| Equil. | LeT | PakG | PakM | India | US |
|---|---|---|---|---|---|
| $E^7_{0.1,1,3}$ | 0.5: attack<br>0.5: none | pros | expand | covert | cut |
| $E^8_{0.1,1,3}$ | 0.25: attack<br>0.75: none | pros | expand | covert | cut |
| $E^9_{0.1,1,3}$ | none | pros | expand | covert | cut |

The rest of the paper is as follows: In Section II, we formally define the instantiation and computation of *well-support multiple $\epsilon$-approximate Nash equilibria*, which are a specific type of the equilibria discussed above and give an algorithm to compute such equilibria. Section III describes the experimental five-player game used to model LeT along with a brief description of the computational system implemented. Section IV summarizes results from computing equilibria from three payoff matrices (created by area experts using open source data) and presents key policy results, together with an assessment of the likelihood that these policies will succeed. Section V describes related work on game-theoretic models of terrorist group behavior as well as past policy recommendations on how the US and India should deal with LeT.

## II. Technical Preliminaries

In this section, we overview the equilibrium concept used by PREVE. We begin by reviewing common game-theoretic models and equilibrium concepts, then briefly discuss the *well-supported multiple $\epsilon$-approximate Nash equilibria* analyzed in this paper, as well as the algorithm used to compute them.

### A. Game Theory Preliminaries

We consider simultaneous multiplayer games. Let $[n] = \{1, 2, \ldots, n\}$ be the set of players and $[m] = \{1, 2, \ldots, m\}$ be the set of actions for each player. Let $\Delta_m$ be the simplex $\{(x_1, x_2, \ldots, x_m) | \sum_{i \in [m]} x_i = 1, x_i \geqslant 0, \forall i \in [m]\}$.

For any player $j$ and $\sigma^j \in \Delta_m$, $\sigma^j$ is a probability distribution function over the set of actions $[m]$; thus, $\sigma^j$ is called a *strategy* for player $j$. If $\sigma^j = (x_1, x_2, \ldots, x_m)$, then $x_i$ is the probability that player $j$ will perform action $i$. When all but one of the $x_i$'s in $\sigma^j$ are 0, then $\sigma^j$ is called a *pure strategy*; otherwise, it is called a *mixed strategy*. In mixed strategies, a player probabilistically chooses which action to take—but note that we will calculate these mixed strategies from the multiple payoff matrices provided by experts. They are not inputs to our algorithms (and so experts do not have to provide them); they are outputs generated by our system.

We use $\Delta$ to denote the set $\Pi_{j=1}^{n} \Delta_m$. Any $\sigma \in \Delta$ is called a *strategy profile* for a game a. If $\sigma = (\sigma^1, \ldots, \sigma^n) \in \Delta$, then $\sigma^j$ denotes the strategy of the player $j$. For convenience, we can represent a strategy profile $\sigma$ as $(\sigma^j, \sigma^{-j})$, where $\sigma^j$ represents the strategy of player $j$ and $\sigma^{-j}$ represents strategies for the rest of the players.

The *payoff* for a player $j$ is a function $u_j : \Delta \mapsto [0, 1]$. In this section, we assume (without loss of generality) that all payoffs are in the unit interval $[0, 1]$. We now define a basic building block of game theory, the Nash equilibrium.

**Definition 1.** A strategy profile $\sigma$ is a *Nash equilibrium* iff:

$$u_j(\sigma^{j\prime}, \sigma^{-j}) \leqslant u_j(\sigma) \quad \forall \sigma^{j\prime} \in \Delta_m, j \in [n]$$

Thus, a strategy profile is a Nash equilibrium if no player has incentive to deviate from his strategy, assuming all other players play their respective strategies. Classical game theory assumes that players are rational. Hence, players can reason about one another and identify the Nash equilibria that are possible and then typically play actions consistent with one such Nash equilibrium. As Schelling [3] observes, a good amount of work may also be invested by players in "prepping" the game so that certain strategy profiles are excluded from being equilibria.

Since such equilibria are notoriously difficult to compute [9], [5], recent work has focused on finding *approximate* Nash equilibria. We use a very well-known notion of an approximate Nash equilibria.

Let $S(\sigma)$, the *support of* a strategy $\sigma \in \Delta_m$, be the set $S(\sigma) = \{i \mid \sigma_i > 0\}$, i.e. the support of $\sigma$ is the set of actions that are executed with nonzero probability. Daskalakis, Goldberg, and Papadimitriou [5] define a well-supported approximate Nash equilibrium as follows.

**Definition 2.** Suppose $0 \leqslant \epsilon \leqslant 1$ is a real number. A strategy profile $\sigma$ is a well-supported $\epsilon$-approximate Nash equilibrium iff:

$$u_j(e_i, \sigma^{-j}) \leqslant u_j(e_l, \sigma^{-j}) + \epsilon \quad \forall \sigma^{j\prime} \in \Delta_m, i \in [m], \\ l \in S(\sigma^j), j \in [n]$$

In other words, for a strategy to be a well-supported $\epsilon$-approximate Nash equilibrium, every player's incentive to deviate from his equilibrium strategy is very small (less than a utility of $\epsilon$).

This definition tries to reduce complexity of computing approximate equilibria by only considering payoffs of actions in the *support* of a strategy profile.

### B. Multiple Payoff (Approximate) Equilibria

In this section, we merge together the ideas of well-supported approximate Nash equilibria and Shapley's vector payoffs so that multiple (conflicting) experts' knowledge of payoffs can be seamlessly handled.

**Definition 3.** If $[n]$ is a set of players, $[m]$ is the set of actions for each player in $[n]$, and $U = (U_1, U_2, \ldots, U_f)$ consists of $f$ ordered sets of payoff functions $U_k = (u_1^k, u_2^k, \ldots, u_n^k), \forall k \in [f]$, then a *simultaneous game with multiple payoff functions* (SGM) is a tuple $G = (n, m, U)$.

Intuitively, an SGM $G$ can be viewed as $f$ different games specified over a set of players, over the same strategy space, with payoff functions for players given by $U_k, k \in [f]$. We refer to these $f$ individual simultaneous games as *constituent games* of $G$. We now merge the ideas of well-supported approximate Nash equilibrium (Def. 2) and vector payoffs.

**Definition 4.** A strategy profile $\sigma$ is a *well-supported multiple $\epsilon$-approximate Nash equilibrium* of an SGM $(n, m, U)$, iff it is a well-supported $\epsilon$-approximate Nash equilibrium for each of its constituent games. That is, for all $k \in [f]$:

$$u_j^k(e_i, \sigma^{-j}) \leqslant u_j^k(e_l, \sigma^{-j}) + \epsilon \quad \forall \sigma^{j\prime} \in \Delta_m, i \in [m], \\ l \in S(\sigma^j), j \in [n]$$

Informally speaking, a well-supported multiple $\epsilon$-approximate Nash equilibrium is an equilibrium that is "close" in payoff for each player to a (Nash or approximate Nash) equilibrium in the constituent game corresponding to each payoff matrix in the SGM. In other words, a well-supported multiple $\epsilon$-approximate Nash equilibrium closely approximates equilibrium situations irrespective of which of the several experts' payoff matrices is used and therefore, it is a robust type of equilibrium.

For notational convenience, in the experimental section of this paper, we will refer to well-supported multiple $\epsilon$-approximate Nash equilibria computed using only $U' \subseteq U$ payoff functions as $(\epsilon, k)$*-equilibria*, where $|U'| = k$. Such equilibria computed with the full set $U$ are simply written as $\epsilon$-*equilibria*.

### C. t-Uniform Strategies

In this section, we define the concept of a uniform strategy. We prove that uniform strategies can be used to compute approximate Nash equilibria for these games when the number of actions is small (which is true in the case of our LeT game where each player has 3 or 4 actions) and use this result to design an algorithm to compute multiple payoff equilibria for such games.

In this and the next section, we focus only on uniform strategies. Uniform strategies provide a tradeoff between simplicity and optimality that may be valuable to the end user. For example, in the LeT game we are studying, the policy prescription: "India should take covert action against LeT with probability 0.0071" may not be very useful for the end user. A simpler policy prescription that is *almost* as good may be a much better option.

We now define a uniform strategy profile.

**Definition 5.** A strategy profile $\sigma = (\sigma^1, \sigma^2, \ldots, \sigma^n)$ is a $t$-uniform strategy profile if, $\forall j \in [n], \forall i \in [m]$:

$$\sigma_i^j \in \left\{ \frac{k}{t} \mid k \in \{0, 1, \ldots, t\} \right\}$$

The parameter $t$ above controls the granularity of distribution on actions in a strategy. A smaller $t$ leads to a coarse-grained, simple strategy whereas a larger $t$ allows a more fine-grained strategy that may be closer to an optimal strategy.

We now prove that a uniform strategy profile can be used to approximate a Nash equilibrium for multiplayer games of low rank which is what our LeT game is. Our proof is by construction. First, we state the following lemmas to help with the main result.

**Lemma 6.** *Let $\alpha$ be a vector of length $m$ such that each element of $\alpha$ is in $[0,1]$. Let $\sigma$ be vector of length $m$. Let $\sigma'$ be a vector such that $|\sigma_i - \sigma'_i| \leqslant \epsilon, \forall i \in [m]$. Then $|\alpha^T \sigma - \alpha^T \sigma'| \leqslant m\epsilon$.*

This lemma is a technical lemma which says that if $\sigma, \sigma'$ are two strategies whose probabilities are almost the same (i.e. differ by at most $\epsilon$) for all actions, then multiplying then by a vector of real numbers also returns two strategies that are almost the same except for a multiplicative factor corresponding to the number of actions $m$ (i.e., they differ by at most $m \cdot \epsilon$).

The following technical lemma is straightforward and says that when we multiply two real-valued vectors of length $n$ whose individual entries are almost equal, then the difference in their products is at most $n \cdot \epsilon$.

**Lemma 7.** *Let $x_1, \ldots, x_n$ be $n$ reals such that $0 \leqslant x_i \leqslant 1, \forall i \in [n]$. Let $x'_1, \ldots, x'_n$ be $n$ reals such that $0 \leqslant x'_i \leqslant 1, |x_i - x'_i| \leqslant \epsilon, \forall i \in [n]$. Then $|\prod_{i \in [n]} x_i - \prod_{i \in [n]} x'_i| \leqslant n\epsilon$.*

The main technical result of this subsection is that if a strategy profile is a well-supported, then there exists a $t$-uniform strategy profile that is a well-supported $\epsilon$-approximate equilibrium with a slightly higher $\epsilon$. However, the simpler lemma below provides the basis for the more complex theorem to follow.

**Lemma 8.** *Let the strategy profile $\sigma = (\sigma^1, \sigma^2, .., \sigma^n)$ be a well-supported $\epsilon$-approximate Nash equilibrium for the given game of rank $k$. Then there exists a $t$-uniform strategy profile $\sigma'$ that is a well-supported $\epsilon + \frac{2(n-1)mk}{t}$-approximate Nash equilibrium.*

Now, we extend this result to the multiple payoff matrix case.

**Theorem 9.** *Let the strategy profile $\sigma$ be a well-supported $\epsilon$-approximate Nash equilibrium for the given SGM, all of whose constituent games are rank $k$ games. Then, there exists a $t$-uniform strategy profile $\sigma'$ that is a well-supported $\epsilon'$-approximate Nash equilibrium, with $\epsilon' = \epsilon + \frac{2(n-1)mk}{t}$.*

### D. Computing Multiple Payoff Equilibria

We now present a grid search algorithm for computing well-supported multiple $\epsilon$-approximate Nash equilibria which is the basis for the code implemented in PREVE leveraging the main theoretical results of Theorem 9 presented in the preceding subsection. This result states that when the number of possible actions for each player in an SGM is small—an assumption which is true in our five-player game and that holds in many real-world games—then a grid search over the space of uniform strategies is an effective method to find a class of well-supported $\epsilon$-approximate equilibria that have the $t$-uniformity property defined above.

The equilibria analyzed in this paper are found via an *exhaustive* grid search over uniform strategy profiles of the SGM, as detailed in Algorithm 1. Uniform strategies are expected to provide good results on general games [10], as well as our own. While Algorithm 1 is an exhaustive search over the space of uniform strategy profiles, we emphasize that this search space is significantly sparser than that of all strategy profiles—which should result in runtime gains when compared to computation methods over the full space. Input parameters to the general algorithm are $\epsilon$, $t$ and payoff functions for the constituent games.

In this paper, payoff functions are given by policy experts. The output of the algorithm is then the set of all $t$-uniform strategy profiles which are well-supported multiple $\epsilon$-approximate Nash equilibria for a given SGM. Before introducing the real-world five-player game and subsequent policy analysis, we present results showing the algorithm's performance and output on generated games.

### E. Scaling Characteristics of Algorithm 1

To test the scaling properties of Algorithm 1, we built a game generator and varied the number of experts (each giving one set of payoff matrices), players, and actions per player. We also varied the granularity factor $t$ when generating $t$-uniform strategies. The framework was implemented in about 700 lines of C++, and the experiments were run on a 4-CPU, 4-core Intel Xeon 3.4GHz machine with 64GB of RAM.

**Algorithm 1:** Exhaustive grid search for equilibria

**Input**: $\epsilon$, $t$, payoff functions $P$ for the SGM
**Output**: $t$-uniform $\epsilon$-equilibria for the SGM
$S \leftarrow$ *Set of all possible $t$-uniform strategies*
$E \leftarrow \varnothing$
$\Sigma \leftarrow \times_{i=1}^{n} S$
**for** $l \in [P]$ **do**
  $E_l \leftarrow \varnothing$
  **for** $\sigma \in \Sigma$ **do**
    $isEquilibrium \leftarrow$ TRUE
    **for** $j \in [n]$ **do**
      **if** $\neg isEquilibrium$ **then** break
      $payoff \leftarrow u_j^l(\sigma)$
      **for** $i \in [m]$ **do**
        $payoff_i \leftarrow u_j^l(e_i, \sigma_{-j})$
        **if** $payoff_i - payoff > \epsilon$ **then**
          $isEquilibrium \leftarrow$ FALSE
          break
    **if** $isEquilibrium$ **then** $E_l \leftarrow E_l \cup \{\sigma\}$
**return** $\bigcap_{i=1}^{|P|} E_i$

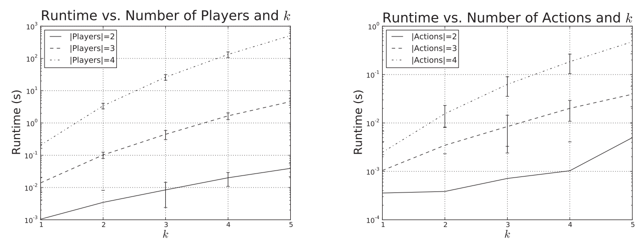


Fig. 1. Runtime as the number of players increases (left) and number of actions increases (right) for $t$-uniform factor $t \in \{1, \ldots, 5\}$.

Figure 1 shows the runtime of Algorithm 1 on generated data as both the number of players and number of actions increase, for varying granularity factors. As expected, increasing the number of players (while holding the number of actions constant) hurts runtime significantly more than increasing the number of actions (while holding the number of players constant). Similarly, increasing the granularity factor $t$ (shown on the x-axis) exponentially increases the number of possible strategy profiles over which the algorithm must iterate, resulting in large runtime increases. Future research would increase the algorithm's equilibrium-generation capabilities to games with many players and many actions. Figure 2 quantifies the relationship between the $\epsilon$-approximation threshold and the percentage of strategy profiles that are equilibria. Intuitively, increasing the slack in the approximation factor $\epsilon$ yields a higher percentage of strategy profiles being equilibria, while increasing the number of potential payoff matrices decreases this percentage of strategy profiles. The rate of increase of this line is highly dependent on the distribution of payoffs to each individual player. With random generation of payoffs, the increase is fairly steady; however, a more structured (e.g., real-world) payoff function would affect this trend.

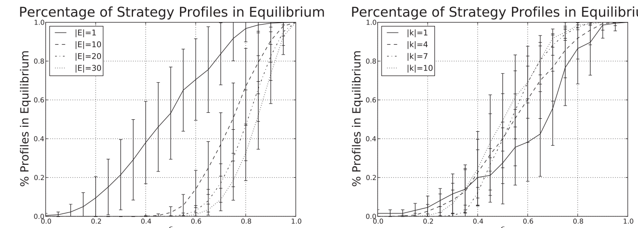


Fig. 2. Percentage of all sets of strategy profiles that are equilibria as the number of experts increases (left) and $t$-uniform factor increases (right), for $\epsilon \in \{0.0, 0.05, \ldots, 1.0\}$.

In the next section, we present just such a study on a real-world five-player game with payoff functions determined by three senior experts.

## III. Game Definition

In this paper, we consider a five-player game with three or four unique actions per player. The players considered are the United States (US), India, Pakistan's government, Pakistan's military, and Lashkar-e-Taiba (LeT). We recall that Table I, presented earlier, gives actions each player can take, and that—in addition to the actions below—each player can take the action `none`, which corresponds to doing nothing.

**US Actions.** The US can take three actions (and `none`).

1) The first is `covert` action against LeT. While we do not suggest specific operations, this action could be implemented in many ways including covert actions to undermine LeT's leaders or covert actions to target LeT training camps. It is clear that the US is capable of such covert action as evidenced by recent events involving a CIA contractor called Raymond Davis who was arrested by the Pakistanis after a shootout in Lahore.
2) The US could also `cut` military and/or development support currently being given to Pakistan. According to the Congressional Research Service, in FY 2010 alone, the US provided $1.727 billion in economic aid to Pakistan in FY2010.[1] In 2012, the US asked Congress for permission to ship almost $3 billion to Pakistan with over half being military aid.[2] Cutting some of this aid is an option the US has long considered, especially in view of US Admiral Mike Mullen's assertions in 2011 about Pakistan's ISI controlling the Haqqani terrorist network which in turn attacked the US embassy in Kabul.[3]
3) The US could also `expand` financial support for Pakistan. Pakistan's educational system and economy are both in shambles and some have argued that additional development assistance would wean young people away from radical elements.

**India's Actions.** As with the US, we study three actions (and `none`) that India might take. Similarly, there are many ways in which India could tactically implement these actions.

1) Like the US, India can also take different forms of `covert` action against LeT using methods similar to those listed above for the US.

[1] See "Pakistan-U.S. Relations: A Summary," by K. Alan Kronstadt of the Congressional Research Service, May 16, 2011.
[2] http://www.foxnews.com/topics/us-aid-to-pakistan-fy2012-request.htm
[3] http://www.nytimes.com/2011/09/23/world/asia/mullen-asserts-pakistani-role-in-attack-on-us-embassy.html?pagewanted=all&_r=0

2) India can also use `coercive diplomacy` in which diplomatic moves are used to coerce Pakistan. For instance, a credible threat can be used to warn Pakistan of the consequences of carrying out certain actions. For coercive diplomacy to be effective, the threat must be made publicly and must be credible [3]. Credible threats could include withholding water by diverting the headwaters of the Indus or by troop movements or simply by ramping up military spending which would place pressure on other parts of the Pakistani economy.
3) A third option we consider is one where India proposes some kind of `peace` initiative to Pakistan, e.g. some additional rights for back and forth movement between India and Pakistan, unifying families in Kashmir who were split up by the partition of Kashmir, and so forth.

**Pakistan Military Actions** We study three possible actions for the Pakistani military, all related to their support for LeT.

1) The Pakistani military could implement a `crackdown` on LeT by arresting LeT members and/or closing down LeT's training camps, shutting down the logistical support for LeT operations in Jammu and Kashmir, and taking steps to interdict LeT-allied organizations like Jamaat-ud-Dawa. Pakistani security has, at times, cracked down on LeT, e.g. after the December 2001 parliament attack and the November 2008 attacks in Mumbai.
2) The Pakistani military could `cut` support to LeT by, e.g., arresting military officers who are illicitly supporting LeT and stopping military training of LeT personnel.
3) The Pakistani military could also `expand` support for LeT, e.g. by increasing its logistical and materiel support as well as financial support.

**Pakistan Government Actions** We consider just two possible actions (in addition to `none`) by the civilian side of the Pakistani government (excluding the military side).

1) The Pakistani government could `prosecute` and arrest LeT personnel, as they have done periodically (though the leaders are usually released shortly thereafter).
2) The Pakistani government could choose to `endorse` LeT's social services program by routing government services through them. LeT runs many social services in Pakistan ranging from ambulances to hospitals, schools, and disaster relief programs.

**Lashkar-e-Taiba's Actions** In the case of LeT, we considered three actions (in addition to the `none` action).

1) LeT could launch a major `attack`. We already know from the November 2008 Mumbai siege that they have the capability and logistical support to execute such attacks.
2) LeT could `hold` attacks (but not major ones), similar to those periodically carried out by them in Kashmir where military and civilian personnel are frequently targeted.
3) LeT could do something dramatic like eliminate its armed wing, give up its weapons, and publicly renounce violence. Though extremely unlikely, this is still worth listing as a possible action.

TABLE IV. STATISTICS ON NUMBER OF $(\epsilon, 2)$ EQUILIBRIA FOUND.

| $n$ | $\epsilon$ | #Eq. found | #Eq. without LeT attacks |
|---|---|---|---|
| 2 | 0 | 252 | 6 |
| 2 | 0.1 | 357 | 6 |
| 2 | 0.2 | 1696 | 9 |
| 2 | 0.3 | 13925 | 42 |

## IV. POLICY ANALYSIS RESULTS

Before presenting the policy implications of the results generated by PREVE, we present a summary of the $(\epsilon, n)$-equilibria we found in Table IV. We limit the equilibria presented to those where LeT does not attack. No such $(\epsilon, 3)$-equilibria were found for $\epsilon \leqslant 0.5$, so we focus on the case when $n = 2$. In the case of mixed equilibria, we list an equilibrium as having no LeT attacks when the probability of LeT attacking (action `attack`) or holding its current set of attacks (action `hold`) is 25% or less.

In the rest of this section, we consider $(\epsilon, 2)$-equilibria, for $\epsilon \in \{0.0, 0.1, 0.2\}$. Though we computed $(\epsilon, 2)$-equilibria for $\epsilon = \{0.3, 0.4, 0.5, \ldots\}$ as well, we note that all of these equilibria involve players giving up 30% or more of their payoffs—something that we think is unlikely.

Of the 252 $(0, 2)$-equilibria, we observed that there were five equilibria in which the US cut aid, India carried out either covert operations against LeT or coercive diplomacy against Pakistan, and the Pakistani military cracked down on LeT. In every one of these situations, LeT either eliminated its armed wing or did nothing, and the Pakistani government either prosecuted LeT or did nothing. Moreover, there are 24 $(0, 2)$-equilibria in which the US cuts aid and India carries out either covert action or coercive diplomacy—and in 5 of these 24 equilibria, LeT either eliminated its armed wing or did nothing. This may suggest that these equilibria give only a $5/24 \approx 21\%$ probability of success even if the US and India join forces to combat LeT. However, the situation is more complex. In our data, we noticed that one expert's payoffs were significantly different from those of the other two. In fact, there were vastly more equilibria between experts #2 and #3 than between experts #1 and #2 or between #1 and #3, suggesting expert #1 was a bit of an outlier. If we only consider experts #2 and #3, then we have a $5/14 \approx 36\%$ chance of getting LeT to stand down if the US cuts aid to Pakistan and India either engages in covert action or coercive diplomacy against Pakistan. Of course, other inducements not considered in this study can be used to get the Pakistani military to crackdown on LeT.

We continued the same analysis of the 357 $(0.1, 2)$-equilibria. There were a total of 23 equilibria where the US cut aid and India acted covertly. Of these, 6 equilibria led to LeT either disbanding its armed wing or doing nothing—good outcomes for peace—leading to a $6/23 \approx 26\%$ probability of success. If we ignored expert #1 (who continued to be an outlier when we considered $(0.1, 2)$-equilibria), then this probability went up to $6/20 = 30\%$ probability of success. Again, when the Pakistani military cracked down on LeT, there was a 100% chance of LeT either eliminating its armed wing or getting rid of terrorism altogether.

When we look at the 1696 $(0.2, 2)$-equilibria, we see a similar pattern. We had a total of 51 $(0.2, 2)$-equilibria, of which LeT cut attacks in 9. There were only 51 of these $(0.2, 2)$-equilibria in which the US cut aid and India took either covert action or engaged in coercive diplomacy. Thus, these two actions by the US and India, if executed in a coordinated fashion, seem to have only a $9/51 \approx 18\%$ probability of success in reducing LeT's attacks. However, we note that when the Pakistani military also cracks down on LeT (in addition to the US and Indian actions just described), there is an $8/9 \approx 89\%$ chance of eliminating LeT's attacks.

## V. Related Work

We split the related work section into two parts: a part dealing with the technical side (game theory, computing), and a part dealing with the social science side.

Though there has been extensive work on the use of game theory for political analysis, almost none of it involves large multiplayer games, and almost none of it involves the use of formal computational methods. The use of game theory to study conflict was pioneered by Schelling [3] who developed a social scientist's view of how 2-player conflicts including terrorism could be studied via game theory. Later, Bueno de Mesquita [11] recounts how he used 2-person games to predict various actions including one of interest in this project, namely that President Obama would not be able to stop Pakistani-based terrorism. Both these efforts and similar efforts focus on two player games; in contrast, the theory of equilibria in multiplayer games with multiple payoff matrices was not described by either of them. Lastly, this paper uses the LeT game proposed in [7]. In contrast to this work, which used only one payoff matrix corresponding to the views of a single expert, we use a multiple payoff matrix model in this paper for which the relevant game theory and the resulting implications for dealing with LeT had to be completely reconsidered.

Ozgul et al. [12] have studied the problem of detecting terror cells in terror networks and proposed a variety of algorithms such as the GDM and OGDM methods. Similarly, Lindelauf et al. [13] have studied the structure of terrorist networks and how they need to maintain sufficient connectivity in order to communicate as well as maintain sufficient disconnectivity in order to stay hidden. They model this tension between communication and covertness via a game-theoretic model. This same intuition led to the concept of covertness centrality [14] in social networks where a statistical (rather than game-theoretic) method is used to predict covert vertices in a network.

Sandler and Enders [15] use the ITERATE data set of terrorist events to discuss how economic methods including both game theory and time-series analysis can be used to propose policies for counter-terrorism. In an earlier survey [16], the same authors specify how game theory might be used to model target selection by terrorists. Major [17] uses a mix of game theory, search, and statistical methods to model terrorism risk. None of these works provide a formal game-theoretic model involving both multiple players and multiple payoff matrices.

On the social science side, Clark [18] was the first to study LeT from a military perspective. He argues that LeT has grown beyond the control of Pakistan and the ISI and that it will continue to grow with help from fringe elements in the Pakistani military establishment. He argues that India can only insulate itself from LeT-backed attacks by diminishing the internal threat posed by the Indian Mujahideen, an Indian group closely affiliated with LeT. Tankel [19] wrote a detailed analysis of LeT based on years of field work and multiple visits to Pakistan to interview both LeT operatives as well as members of Pakistan's Inter Services Intelligence organization. He provides a wonderful insight into LeT's origins, ideology, and operational structure, but does not include a policy analytics section specifically saying how to deal with the menace posed by LeT. John's excellent volume [20] on the same topic provides another in-depth study of LeT but does not propose policies on how the US and/or India can collectively help reduce LeT attacks. Subrahmanian et al. [2] performs a data mining study of LeT involving 770 variables that are analyzed via data mining algorithms to learn the conditions under which LeT executes various types of attacks. It goes on to consider the problem of shaping the behavior of LeT by using abductive inference models. Another excellent recent book on Pakistan in general by Bruce Riedel [21], a former top CIA official who advised the last five US presidents on relations with India and Pakistan, lays much of the blame for terrorism out of Pakistan (including LeT terrorism) squarely at the doorstep of the Pakistani intelligence agency but does not address LeT attacks in particular.

## VI. Conclusion

Pakistan is widely recognized as being one of the biggest threats to global security today because of several factors: (i) its nuclear arsenal, (ii) the large milieu of violent terrorist and extremist groups in the area with close ties to Pakistani intelligence, (iii) tensions with India, and (iv) a collapsing economy. In this paper, we have focused primarily on Pakistan-India relations, which India views primarily through the lens of terrorist acts in India that are backed by the Pakistani military and are usually operationally executed by LeT and/or its allies, like the Indian Mujahideen.

In this paper, we present PREVE, a set of algorithms based on multiplayer game theory that extends a game developed earlier in [7] to the case where there are multiple payoff matrices that reflect differing opinions of different experts. All game theory requires a payoff matrix showing the values of various strategies (combinations of actions, one by each player) and the resulting equilibria identified are very sensitive to the payoff matrix. But experts do not always agree. In order to provide policy recommendations that are robust with respect to the often differing views of multiple experts, we develop a theory that merges the concepts of $\epsilon$-approximate equilibria and Lloyd Shapley's vector equilibria [22]. We develop an algorithm to compute equilibria in accordance with this theory. As a consequence, the resulting equilibria are much more robust to variations than the equilibria developed in [7] that are very sensitive to minor changes in the payoff matrix.

The PREVE theory, framework, and code have been developed in order to help policymakers with an interest in peace in South Asia determine the best ways for the parties involved to move forward in order to reduce the threat of Lashkar-e-Taiba. Though we applied PREVE only to LeT in this paper, the theory is general and can be applied to any set of actors with any set of actions as long as one or more payoff matrices are available. In this paper, area experts used *open source* data to create payoff matrices for our five-player game.

From a public policy perspective, the results of this paper clearly indicate three things.

1) The US must cut aid to Pakistan. There are no equilibria where LeT behaves well where the US is providing aid to Pakistan. However, we do not have a recommendation for exactly how much this cut should be, only that cuts need to be made.
2) India must engage in additional covert action against LeT and its allies and/or coercive diplomacy towards Pakistan. By cutting aid, the US would intuitively increase political and economic pressure on the Pakistani establishment, leading to a potential loss of support for the Pakistani military leadership amongst the Pakistani people. By engaging in covert action, India would put operational constraints on LeT, making attacks harder by "taking the fight to them" as the US has done against Al-Qaeda. By taking steps towards coercive diplomacy, India would concurrently increase pressure on the Pakistani government and military, complementing the US aid cuts proposed.
3) Despite the measures listed in items (1) and (2) above, chances of success at inducing LeT to eliminate its armed wing and/or do nothing are not great, roughly ranging from 20% to 40%. The key element is getting the Pakistani military to crack down on LeT, in conjunction with US cuts on aid to Pakistan and covert action/coercive diplomacy by India. In this case, the probability of success skyrockets to over 80%. The key question is how to induce the Pakistani military to crack down on LeT. This is the subject of our next study that will examine the deep social, political, economic, and jihadist links that the Pakistani military has and the pressures that might induce them to crack down on extremist elements, many of whom they currently support.

PREVE is a codebase, not an operational system. Top politicians and policymakers are busy and are often more interested in white papers addressing their problem than learning how to use software systems. In our case, PREVE has been used to generate these results and then generate a report interpreting the results for policymakers. The results of this study have been disclosed to top government officials in both the US and Indian government. There is significant interest in continuing these studies.

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## REFERENCES

[1] A. Mannes, "Qualitative analysis & computational techniques for the counter-terror analyst," *Handbook of Computational Approaches to Counterterrorism*, pp. 83–98, 2013.

[2] V. Subrahmanian, A. Mannes, A. Sliva, J. Shakarian, and J. P. Dickerson, *Computational Analysis of Terrorist Groups: Lashkar-e-Taiba*. Springer New York, 2012.

[3] T. Schelling, *The Strategy of Conflict*. Harvard University Press, 1960.

[4] L. Shapley and F. Rigby, "Equilibrium points in games with vector payoffs," *Naval Research Logistics Quarterly*, vol. 6, no. 1, pp. 57–61, 1959.

[5] C. Daskalakis, P. W. Goldberg, and C. H. Papadimitriou, "The complexity of computing a nash equilibrium," in *STOC*, 2006.

[6] R. Myerson, *Game Theory: Analysis of Conflict*. Harvard University Press, 1997.

[7] J. P. Dickerson, A. Mannes, and V. Subrahmanian, "Dealing with Lashkar-e-Taiba: A multi-player game-theoretic perspective," in *Open Source Intelligence and Web Mining (OSINT)*, Athens, Greece, 2011.

[8] S. Tankel, *Lashkar-e-Taiba: Past Operations and Future Prospects*. New America Foundation, 2011.

[9] X. Chen and X. Deng, "Settling the complexity of two-player Nash equilibrium," in *FOCS*, 2006.

[10] R. Lipton, E. Markakis, and A. Mehta, "Playing large games using simple strategies," in *EC'03*. ACM, 2003, pp. 36–41.

[11] B. B. de Mesquita, *The Predictioneer's Game: Using the Logic of Brazen Self-Interest to See and Shape the Future*. Random House, 2010.

[12] F. Ozgul, J. Bondy, and H. Aksoy, "Mining for offender group detection and story of a police operation," in *Proceedings of the Sixth Australasian Conference on Data Mining and Analytics*. Australian Computer Society, 2007.

[13] R. Lindelauf, P. Borm, and H. Hamers, "The influence of secrecy on the communication structure of covert networks," *Social Networks*, vol. 31, no. 2, pp. 126–137, 2009.

[14] M. Ovelgönne, C. Kang, A. Sawant, and V. S. Subrahmanian, "Covertness centrality in networks," in *ASONAM*, 2012, pp. 863–870.

[15] T. Sandler and W. Enders, "An economic perspective on transnational terrorism," *European Journal of Political Economy*, vol. 20, no. 2, pp. 301–316, 2004.

[16] W. Enders and T. Sandler, "Terrorism: Theory and applications," *Handbook of defense economics*, vol. 1, pp. 213–249, 1995.

[17] J. A. Major, "Advanced techniques for modeling terrorism risk," *Journal of Risk Finance, The*, vol. 4, no. 1, pp. 15–24, 2002.

[18] R. Clarke, *Lashkar-i-Taiba: The Fallacy of Subservient Proxies and the Future of Islamist Terrorism in India*. Strategic Studies Institute, US Army War College, 2010.

[19] S. Tankel, *Storming the World Stage: The Story of Lashkar-e-Taiba*. Columbia University Press, 2011.

[20] W. John, *Caliphate's Soldiers: The Lashkar-e-Tayyeba's Long War*. Amaryllis Press and the Observer Research Foundation, 2011.

[21] B. O. Riedel, *Deadly embrace: Pakistan, America, and the future of the global Jihad*. Brookings Institution Press, 2012.

[22] L. Shapley and F. Rigby, "Equilibrium points in games with vector payoffs," *Naval Research Logistics Quarterly*, vol. 6, no. 1, pp. 57–61, 1959.